Ej. 16122015 RESOLUCIÓN CORTA.

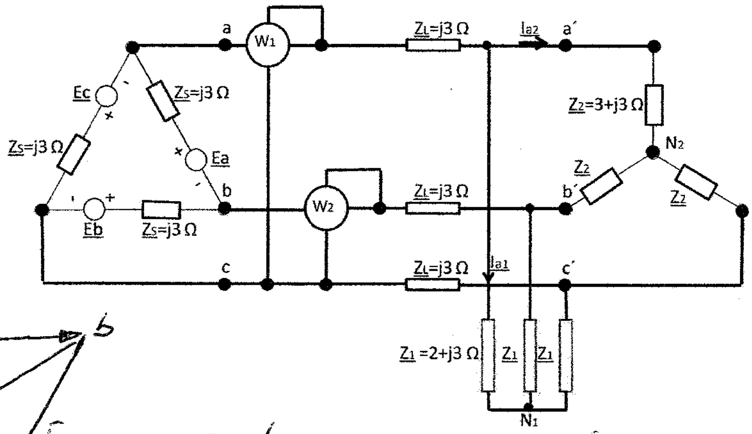
En el circuito trifásico de la figura. Determinar:

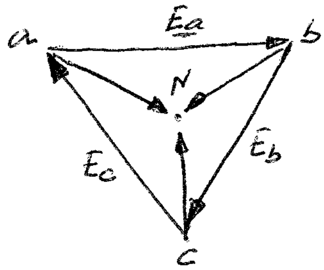
1.- Las corrientes $Ia_1$ e $Ia_2$

2.- Indicaciones de los vatímetros $W_1$ y $W_2$

3.- Calcular la potencia compleja que inyecta al sistema, entre los bornes a y b, del generador *Ea.*

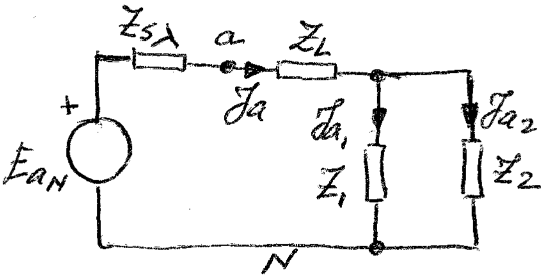
Tensiones de las fuentes: $\underline{Ea} = 398{,}37\angle 0°$; $\underline{Eb} = 398{,}37\angle -120°$ V, $\underline{Ec} = 398{,}37\angle 120°$ V




Sistema equilibrado $\Rightarrow N_1 = N_2 = N$

$$E_{aN} = \left(\frac{E_a}{\sqrt{3}}\right)\angle -30° = 230\angle -30° \text{ V}$$




$$Z_{s\lambda} = \frac{Z_{s\Delta}}{3} = \frac{j3}{3} = j1$$

$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2+j3)(3+j3)}{(2+j3)+(3+j3)} = \frac{-3+j15}{5+j6} = 1{,}959\angle 51{,}12° \ \Omega$$
$$= 1{,}23 + j1{,}525$$

$$Z_{eq} = Z_{s\lambda} + Z_L + Z_p = j1 + j3 + 1{,}23 + j1{,}525 = 5{,}66\angle 77{,}45° \ \Omega$$

1)

$$I_a = \frac{E_{aN}}{Z_{eq}} = \frac{230\angle -30°}{5{,}66\angle 77{,}45} = 40{,}64\angle -107{,}45° \text{ A}$$

$$I_{a1} = \frac{I_a Z_2}{Z_1 + Z_2} = \frac{40{,}64\angle -107{,}45}{5+j6}(3+j3) = 22{,}06\angle -112{,}64° \text{ A}$$

$$I_{a2} = \frac{I_a Z_1}{Z_1 + Z_2} = \frac{40{,}64\angle -107{,}45}{5+j6}(2+j3) = 18{,}78\angle -101{,}34 \text{ A}$$

2)

$$S_{Z1} = I_{a1}^2 (2+j3) = 22{,}06^2 (2+j3) = 973{,}29 + j1459{,}93$$
$$S_{Z2} = I_{a2}^2 (3+j3) = 18{,}78^2 (3+j3) = 1.058{,}07 + j1.058{,}07$$
$$S_{ZL} = I_a^2 \cdot Z_L = 40{,}64^2 \cdot 3j = j4.954{,}83$$

$$P_T = 3(973{,}29 + 1.058{,}07) = 6.094{,}08 \text{ W} = W_1 + W_2$$
$$Q_T = 3(1.459{,}93 + 1.058{,}07 + 4.954{,}83 = 22.418{,}49 \text{ VAr} = \sqrt{3}(W_1 - W_2)$$

$$\left.\begin{array}{l} W_1 + W_2 = 6.094{,}08 \\ W_1 - W_2 = 12.943{,}32 \end{array}\right\} \begin{array}{l} 2W_1 = 19.037{,}40 \Rightarrow \underline{W_1 = 9.518{,}7 \text{ W}} \\ W_2 = W_1 - 12.943{,}32 = \underline{-3.424{,}62 \text{ W}} \end{array}$$

3.- La potencia que se genera entre a y b será la tercera parte (monofásica) de la que consumen las cargas y la línea (Teorema de Boucherot).

$$S_T = 6.094,08 + j22.418,49$$

$$S_m = 2.031,36 + j7472,83$$

Si se considera positiva la potencia consumida, entonces la potencia generada será negativa

$$S_g = -2031,36 - j7472,83 = P_g + jQ_g$$

Ej. 16122015 RESOLUCIÓN LARGA

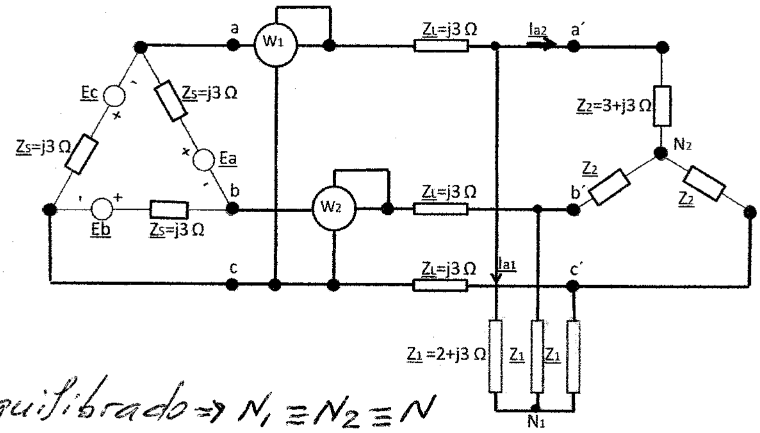
En el circuito trifásico de la figura. Determinar:

1.- Las corrientes $I_{a1}$ e $I_{a2}$

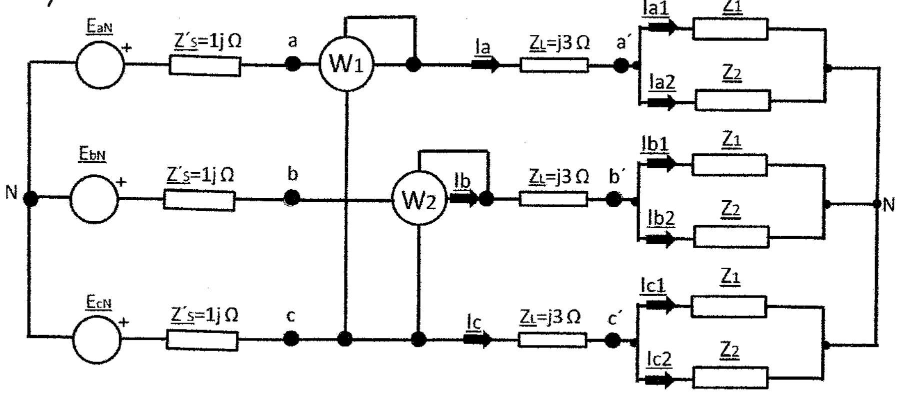
2.- Indicaciones de los vatímetros $W_1$ y $W_2$

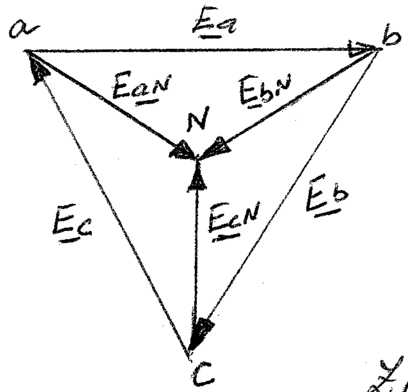
3.- Calcular la potencia compleja que inyecta al sistema, entre los bornes a y b, del generador *Ea.*

Tensiones de las fuentes: Ea =398,37/0º; Eb=398,37/-120º V, Ec=398,37/120º V




• Sistema equilibrado ⇒ $N_1 \equiv N_2 \equiv N$



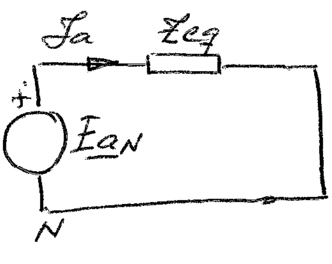




$$\left.\begin{aligned} \underline{E}_{aN} &= \left(\frac{\underline{E}_a}{\sqrt{3}}\right)\angle{-30^\circ} = 230\angle{-30^\circ}\ V \\ \underline{E}_{bN} &= \underline{E}_{aN}\angle{-120^\circ} = 230\angle{-150^\circ}\ V \\ \underline{E}_{cN} &= \underline{E}_{aN}\angle{120^\circ} = 230\angle{90^\circ}\ V \end{aligned}\right\}$$

$$\underline{Z}_p = \frac{\underline{Z}_1\,\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} = \frac{(2+j3)(3+j3)}{(2+j3)+(3+j3)} = \frac{-3+j15}{5+j6} = 1{,}959\angle{51{,}12^\circ}\ \Omega = 1{,}23 + j\,1{,}525$$

$$\underline{Z}'_s = \underline{Z}_{s\lambda} = \frac{\underline{Z}_{s\Delta}}{3} = \frac{j3}{3} = j1$$

$$\underline{Z}_{eq} = \underline{Z}'_s + \underline{Z}_L + \underline{Z}_p = j1 + j3 + 1{,}23 + j1{,}525 = 5{,}66\angle{77{,}45^\circ}\ \Omega$$




$$\underline{I}_a = \frac{\underline{E}_{aN}}{\underline{Z}_{eq}} = \frac{230\angle{-30^\circ}}{5{,}66\angle{77{,}45^\circ}} = 40{,}64\angle{-107{,}45^\circ}\ A$$

$$\underline{I}_b = \frac{\underline{E}_{bN}}{\underline{Z}_{eq}} = \frac{230\angle{-150^\circ}}{5{,}66\angle{77{,}45}} = 40{,}64\angle{-227{,}45^\circ}\ A = 40{,}64\angle{132{,}55^\circ}$$

$$\underline{I}_c = \frac{\underline{E}_{cN}}{\underline{Z}_{eq}} = \frac{230\angle{90^\circ}}{5{,}66\angle{77{,}45^\circ}} = 40{,}64\angle{12{,}55^\circ}\ A$$

$$I_{a1} = \frac{I_a Z_2}{Z_1 + Z_2} = \frac{40{,}64 \angle{-107{,}45}}{5+j6}(3+j3) = \frac{40{,}64\angle{-107{,}45}}{7{,}81\angle{50{,}19}} \cdot 4{,}24\angle{45} = 22{,}06 \angle{-112{,}64°}\ A$$

$$I_{a2} = \frac{I_a Z_1}{Z_1 + Z_2} = \frac{40{,}64\angle{-107{,}45}}{5+j6}(2+j3) = \frac{40{,}64\angle{-107{,}45}}{7{,}81\angle{50{,}19}} \cdot 3{,}61\angle{56{,}30} = 18{,}78\angle{-101{,}34°}\ A$$

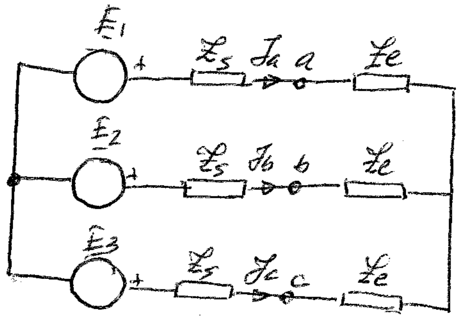
Realmente las corrientes $I_{b1}, I_{b2}, I_{c1}$ e $I_{c2}$ no sería necesario calcularlas, solo habría que tener en cuenta los desfases con respecto a $I_{a1}$ e $I_{a2}$.

$$I_{b1} = \frac{I_b \cdot Z_2}{Z_1 + Z_2} = \frac{40.64\angle{132{,}55}}{7{,}81\angle{50{,}19}} \cdot 4{,}24\angle{45} = 22{,}06 \angle{127{,}36°}\ A$$

$$I_{b2} = \frac{I_b \cdot Z_1}{Z_1 + Z_2} = \frac{40{,}64\angle{132{,}55}}{7{,}81\angle{50{,}19}} \cdot 3{,}61\angle{56{,}30} = 18{,}78 \angle{138{,}66°}\ A$$

$$I_{c1} = \frac{I_c \cdot Z_2}{Z_1 + Z_2} = \frac{40{,}64\angle{12{,}55}}{7{,}81\angle{50{,}19}} \cdot 4{,}25\angle{45} = 22{,}06\angle{7{,}36°}\ A$$

$$I_{c2} = \frac{I_c \cdot Z_1}{Z_1 + Z_2} = \frac{40{,}64\angle{12{,}55}}{7{,}81\angle{50{,}19}} \cdot 3{,}61\angle{56{,}30} = 18{,}78\angle{18{,}66°}\ A$$

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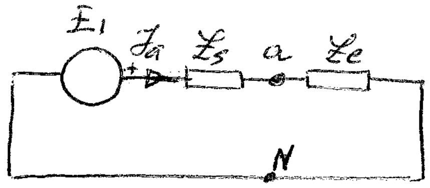


$$Z_e = Z_L + Z_p = j3 + 1{,}23 + j1{,}525 = 1{,}23 + j4{,}525 = 4{,}69\angle{74{,}79}$$

$$U_{ac} + Z_e I_c - Z_e I_a = 0 \Rightarrow U_{ab} = Z_e(I_a - \cancel{I_c})$$

2.- $U_{ac} = 4{,}69\angle{74{,}79} \cdot 40{,}64(1\angle{-107{,}45} - 1\angle{12{,}55})$

$= 190{,}60\angle{74{,}79}(-0{,}2999 - j0{,}954 - 0{,}976 - j0{,}2173)$

$= 190{,}60\angle{74{,}79}(-1{,}2759 - j1{,}1713) = 190{,}60\angle{74{,}79} \cdot 1{,}732\angle{-137{,}45}$

$= 330{,}12\angle{-62{,}66}\ V$

$S_1 = U_{ac} \cdot I_a^* = 330{,}12\angle{-62{,}66} \cdot 40{,}64\angle{107{,}45} = 13.416{,}08\angle{44{,}79}$

$= 9521{,}31 + j\,9.451{,}77 = P_1 + jQ_1 \qquad W_1 = 9.521{,}31\ W$

$U_{bc} = Z_e(I_b - I_c) = 4{,}69\angle{74{,}79} \cdot 40{,}64(1\angle{132{,}55} - 1\angle{12{,}55})$

$= 190{,}60\angle{74{,}79} \cdot (-0{,}6762 + j0{,}7367 - 0{,}976 - j0{,}2173)$

$= 190{,}60\angle{74{,}79}(-1{,}6522 + j0{,}5194)$

$= 190{,}60\angle{74{,}79} \cdot 1{,}732\angle{162{,}55} = 330{,}12\angle{237{,}34}$

$= 330{,}12\angle{-122{,}66°}\ V$

$$S_2 = U_{bc} \cdot I_b^* = 330{,}12 \angle{-122{,}66^\circ} \cdot 40{,}64 \angle{227{,}45} = 13.416{,}08 \angle{104{,}79}$$

$$= -3.424{,}82 + j\,12.971{,}58 = P_2 + jQ_2 \Rightarrow \underline{W_2 = -3.424{,}82\ W}$$

3.- $U_{ab} = Z_e (I_a - I_b) = 4{,}69 \angle{74{,}79} \cdot 40{,}64 (1 \angle{-107{,}45} - 1 \angle{132{,}55})$

$$= 190{,}60 \angle{74{,}79} (-0{,}2999 - j\,0{,}954 + 0{,}6762 - j\,0{,}7367)$$

$$= 190{,}60 \angle{74{,}79} (0{,}3763 - j\,1{,}6907) = 190{,}60 \angle{74{,}79} \cdot 1{,}732 \angle{-77{,}45}$$

$$= 330{,}12 \angle{-2{,}66^\circ}\ V$$

$$U_{ab} - E_a - Z_s \cdot I_i = 0 \quad \Rightarrow \quad I_i = \frac{U_{ab} - E_a}{Z_s}$$

$$I_i = \frac{330{,}12 \angle{-2{,}66} - 398{,}37 \angle{0}}{3 \angle{90^\circ}} = 110{,}04 \angle{-92{,}66} - 132{,}79 \angle{-90} \Rightarrow$$

$$\Rightarrow I_i = -5{,}11 - j\,109{,}92 + j\,132{,}79 = -5{,}11 + j\,22{,}87 = 23{,}43 \angle{102{,}60^\circ}$$

$$S_{ab} = U_{ab} \cdot I_i^* = 330{,}12 \angle{-2{,}66} \cdot 23{,}43 \angle{-102{,}60} = 7.734{,}71 \angle{-105{,}26}$$

$$= -2035{,}77 - j\,7.462 = P_g + jQ_g$$