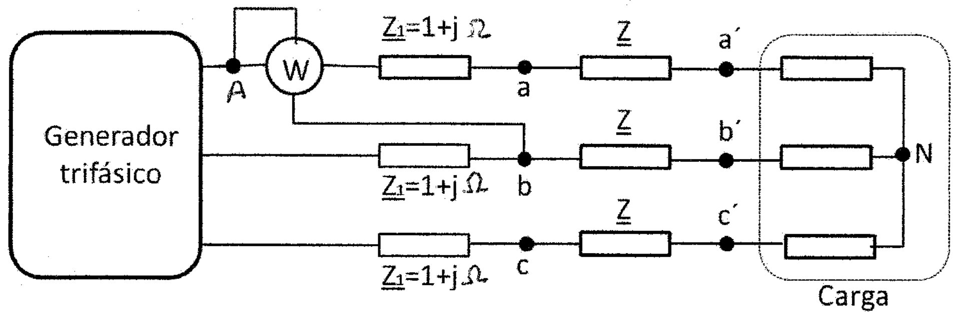
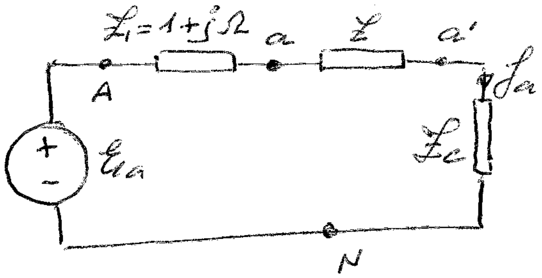
En la figura se representa un sistema trifásico equilibrado de secuencia directa. El valor de las tensiones $\underline{U}_{aN}$ y $\underline{U}_{a'N}$ es el mismo e igual a 100 V. La carga absorbe una potencia activa trifásica de $3000\sqrt{2}$ W, con un factor de potencia $1/\sqrt{2}$ capacitivo.




Se sabe, además, que el circuito pasivo que queda a la derecha de los terminales a,b,c tiene un factor de potencia $\sqrt{3}/2$ inductivo.

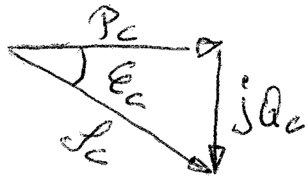
Tomando $\underline{U}_{a'N}$ como origen de fases, hallar el valor de las impedancias complejas $\underline{Z}$ y la indicación del vatímetro.




$|\underline{U}_{aN}| = |\underline{U}_{a'N}| = 100\ V$

$P_c = 3000\sqrt{2}\ W$

$\cos\varphi_c = \frac{1}{\sqrt{2}}\ cap \Rightarrow \varphi_c = 45^\circ$




$Q_c = P_c \operatorname{tg}\varphi_c = -3000\sqrt{2}\ VAr$

$\underline{Z}_{eq} = \underline{Z} + \underline{Z}_c = |\underline{Z}_{eq}| \angle \varphi_{eq}$

$\cos\varphi_{eq} = \frac{\sqrt{3}}{2} \Rightarrow \varphi_{eq} = 30^\circ$

$\underline{S}_c = 3\,\underline{U}_{a'N}\,\underline{I}_a^*$; $\quad \underline{I}_a = \frac{\underline{U}_{a'N}}{\underline{Z}_c} = \underline{U}_{a'N}\,\underline{Y}_c$; $\quad \underline{S}_c^* = 3\,\underline{U}_{a'N}\,\underline{I}_a$

$\underline{S}_c^* = 3\,U_{a'N}^2\,\underline{Y}_c \Rightarrow \underline{Y}_c = \frac{\underline{S}_c^*}{3\,U_{a'N}^2} = \frac{P_c - jQ_c}{3\,U_{a'N}^2} = \frac{3000\sqrt{2} + j3000\sqrt{2}}{3 \cdot 100^2}$

$\Rightarrow \underline{Y}_c = \frac{\sqrt{2}}{10}(1+j) = \frac{1}{5}\angle 45^\circ\ S$; $\quad \underline{Z}_c = \frac{1}{\underline{Y}_c} = 5\angle -45^\circ$

$\underline{U}_{a'N} = \underline{Z}_c\,\underline{I}_a = \underline{Z}_c \cdot \frac{\underline{U}_{aN}}{\underline{Z} + \underline{Z}_c} = \frac{\underline{Z}_c}{\underline{Z}_{eq}}\,\underline{U}_{aN}$

$|\underline{U}_{aN}| = |\underline{U}_{a'N}| \Rightarrow |\underline{Z}_{eq}| = |\underline{Z}_c|$

$|\underline{Z}_{eq}| = 5$

$\underline{Z}_{eq} = 5\angle 30^\circ$ $\qquad \underline{Z} = \underline{Z}_{eq} - \underline{Z}_c = 5\angle 30 - 5\angle -45 \Rightarrow$

$\underline{Z} = 5(0{,}866 + j0{,}5) - 5(0{,}707 - j0{,}707) =$

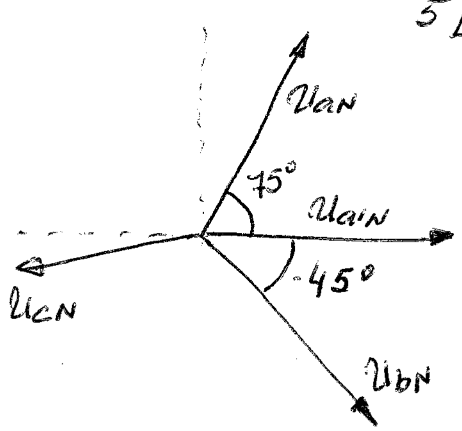
$= 0{,}7945 + j\,6{,}035 = \underline{6{,}087\angle 82{,}5^\circ}$

$$\underline{U}_{a'N} = 100 \angle 0^\circ \text{ V}$$

$$I_a = \frac{U_{a'N}}{Z_c} = \frac{100\angle 0^\circ}{5\angle -45} = 20 \angle 45^\circ \text{ A}$$

$$\underline{U}_{aN} = \frac{\underline{Z} + \underline{Z}_c}{\underline{Z}_c} \cdot \underline{U}_{a'N} = \frac{6.088\angle 82.5 + 5\angle -45^\circ}{5\angle -45^\circ} \cdot 100\angle 0^\circ$$

$$= \frac{4{,}3245 + j\,2{,}5055}{5\angle -45^\circ} \cdot 100\angle 0^\circ = \frac{5\angle 30^\circ}{5\angle -45^\circ} \cdot 100\angle 0^\circ = 100 \angle 75^\circ$$




$$\underline{U}_{bN} = 100\angle -45^\circ = 50\sqrt{2}\,(1-j) \text{ V}$$

$$\underline{U}_{AN} = \underline{U}_{aN} + (1+j)\cdot I_a$$

$$= 100\angle 75^\circ + (1+j)\cdot 20\angle 45^\circ$$

$$= 100\angle 75^\circ + \sqrt{2}\angle 45^\circ \cdot 20\angle 45^\circ$$

$$= 100\angle 75^\circ + 20\sqrt{2}\angle 90^\circ$$

$$\underline{U}_{Ab} = \underline{U}_{AN} - \underline{U}_{bN} = 100\angle 75 + 20\sqrt{2}\angle 90^\circ - 100\angle -45^\circ$$

$$= 25{,}88 + 96{,}59\,j + j\,28{,}28 - 70{,}71 + j\,70{,}71$$

$$= -44{,}83 + j\,195{,}55 = 200{,}62 \angle -77{,}09^\circ$$

$$W_{A(Ab)} = \text{real}\{\underline{U}_{Ab} \cdot I_a^*\} = 200(2 + 5\sqrt{3}) = 2137{,}6 \text{ W}$$

$$\{U_{AB}\, I_a^*\} = 20\angle -45^\circ \cdot 200{,}62\angle -77{,}09 = 4.012{,}4 \angle -122{,}09$$

$$= -2.131{,}6 - j\,3.399{,}36$$

$$\underline{W = 2.131{,}6 \text{ W}}$$